Research article

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Circular Rydberg states of helium atoms or helium-like ions in a high-frequency laser field

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Abstract: In the literature, there were studies of Rydberg states of hydrogenic atoms/ions in a high-frequency laser field. It was shown that the motion of the Rydberg electron is analogous to the motion of a satellite around an oblate planet (for a linearly polarized laser field) or around a (fictitious) prolate planet (for a circularly polarized laser field): it exhibits two kinds of precession – one of them is the precession within the orbital plane and another one is the precession of the orbital plane. In this study, we study a helium atom or a helium-like ion with one of the two electrons in a Rydberg state, the system being under a high-frequency laser field. For obtaining analytical results, we use the generalized method of the effective potentials. We find two primary effects of the high-frequency laser field on circular Rydberg states. The first effect is the precession of the orbital plane of the Rydberg electron. We calculate analytically the precession frequency and show that it differs from the case of a hydrogenic atom/ion. In the radiation spectrum, this precession would manifest as satellites separated from the spectral line at the Kepler frequency by multiples of the precession frequency. The second effect is a shift of the energy of the Rydberg electron, also calculated analytically. We find that the absolute value of the shift increases monotonically as the unperturbed binding energy of the Rydberg electron increases. We also find that the shift has a nonmonotonic dependence on the nuclear charge Z: as Z increases, the absolute value of the shift first increases, then reaches a maximum, and then decreases. The nonmonotonic dependence of the laser field-caused energy shift on the nuclear charge is a *counterintuitive* result.

Keywords: circular Rydberg states, helium atoms, helium-like ions, high-frequency laser field, precession of the orbital plane, energy shift

1 Introduction

The previous studies [1–3] focused on hydrogenic atoms/ions in a high-frequency laser field. In particular, ref. [2,3] focused on Rydberg states in the classical description. Ref. [2,3], by "high frequency," mean that the laser frequency ω is much greater than the Kepler frequency $\omega_K = m_e e^4/(n^3\hbar^3)$ of the highly excited hydrogen atom: $\omega \gg \omega_K$. Here, m_e and e are the electron mass and charge, respectively; $n \gg 1$ is the principal quantum number. In this situation, the laser field and the Rydberg atom can be considered as the fast and slow subsystems, respectively, thus allowing the analytical treatment of the problem. In particular, the authors of ref. [2] generalized Kapitza's method of effective potentials [4,5].

In ref. [1–3], it was revealed that this fundamental problem exhibits a rich physics. When the laser field is linearly polarized or circularly polarized, the system has the axial symmetry, so that the square of the angular momentum M^2 should not be conserved (only its projection M_z on the axis of the symmetry is conserved). However, in ref. [1–3], it was shown that M^2 is approximately conserved in this situation, so that there is an approximate algebraic symmetry higher than the geometrical symmetry.

In addition, when the laser field is linearly polarized or circularly polarized, the system has celestial analogies. Namely, in the linearly polarized laser field, the motion of the Rydberg electron is analogous to the motion of a satellite around an oblate planet: it exhibits two kinds of precession – one of them is the precession within the orbital plane and another one is the precession of the orbital plane. In the circularly polarized laser field, the motion of the Rydberg electron is analogous to the motion of a satellite around a (fictitious) prolate planet: it also exhibits the same two kinds of precession.

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In this article, we study a helium atom or a helium-like ion with one of the two electrons in a Rydberg state, the system being under a high-frequency laser field. For obtaining analytical results, we use the generalized method of the effective potentials from previous studies [2,6] and in the previously published book [7].

Then, we focused at circular Rydberg states.[1] We show that the high-frequency laser field causes the precession of the orbital plane of the Rydberg electron. We calculate analytically the precession frequency and demonstrate that it differs from the case of a hydrogenic atom/ion. In the radiation spectrum, this precession would manifest as satellites separated from the spectral line at the Kepler frequency by multiples of the precession frequency.

We also show that the high-frequency laser field also causes a red shift of the energy of the Rydberg electron. We calculate analytically this energy shift and study its dependence on parameters of the system.

2 New results

We study a He-like atom or ion in a high-frequency laser field. The atom/ion has the inner electron in state 1s and the highly excited (Rydberg) outer electron. The potential Φ of a quasinucleus consisting of the nucleus Z and a spherically symmetric charge distribution corresponding to the inner electron in state 1s is expressed as follows (see, e.g., ref. [39]):

$$\Phi(r) = \frac{Z-1}{r} + \left(Z\mu + \frac{1}{r}\right)e^{-2Z\mu r}, \tag{1}$$

where $\mu = M_n m_e/(M_n + m_e)$ is the reduced mass of the pair "nucleus Z – electron" (M_n is the nuclear mass and m_e is the electron mass) and r is the distance from the center of symmetry to the electron. In this study, we use atomic units $\hbar = e = m_e = 1$. The atom is subjected to a high-frequency laser field of amplitude F and frequency ω. For Rydberg electrons, a classical or semi-classical treatment is appropriate.

2.1 Linear polarization of the laser field

First, we consider the case of the linear polarization of the laser field. The semi-classical Hamiltonian for the outer (Rydberg) electron in this configuration can be represented in the following form:

$$\begin{aligned} H &= H_0 + zF\cos\omega t, \\ H_0 &= \frac{1}{2\mu_1}\left(p_r^2 + \frac{p_\theta^2}{r^2} + \frac{p_\varphi^2}{r^2\sin^2\theta}\right) - \Phi(r), \end{aligned} \tag{2}$$

where $\mu_1 = m_e(M_n + m_e)/(M_n + 2m_e)$ is the reduced mass of the pair "nucleus Z with the inner electron – outer electron," the z-axis is in the direction of the laser field $\mathbf{F}$, (r, θ, φ) are the spherical coordinates of the electron, F is the magnitude, and ω is the frequency of the laser field. Both μ and μ_1 are very close to unity: their physical values lie in the range between 0.999864 (for helium, $Z = 2$) and 0.999998 (for He-like oganesson, $Z = 118$). For the systems in a high-frequency field, when the frequency of the field is much greater than the highest frequency of the unperturbed system, it is appropriate to use the formalism of the effective potentials [2,4–7]. As a result, the Hamiltonian H_0 acquires a time-independent term. The zeroth-order effective potential is expressed as follows:

$$U_0 = \frac{1}{4\omega^2}[V,[V,H_0]] = \frac{F^2}{4\mu_1\omega^2}, \tag{3}$$

where $V = zF$, $[P, Q]$ are the Poisson brackets, which is a coordinate-independent energy shift that does not affect the dynamics of the system. The first nonvanishing effect on the dynamics of the system originates from the first-order effective potential:

$$\begin{aligned} U_1 &= \frac{1}{4\omega^4}[[V,H_0],[[V,H_0],H_0]] \\ &= -\frac{F^2(Z\mu)^3}{8\mu_1^2\omega^4x^3}((Z-1)(1+3\cos 2\theta) \\ &\quad + ((1+2x)(1+2x^2) \\ &\quad + (3+6x+6x^2+4x^3)\cos 2\theta)e^{-2x}), \end{aligned} \tag{4}$$

where we use the notation

$$x = \mu Z r, \tag{5}$$

so that the effective potential energy of the electron is expressed as follows:

1 Circular Rydberg states (CRS) have been studied theoretically and experimentally by numerous authors – see, e.g., papers [8–35] and references therein. The importance of studying CRS is threefold. First, experiments on cold Rydberg atoms and on inhibited spontaneous emission involve CRS because they are characterized by anisotropic cross section of collisions and by long radiative lifetimes [36,37]. Second, for quantal coherent states, CRS are their classical counterparts. Third, the classical description of CRS serves as the primary term in the quantal method employing the expansion in terms of the inverse value of the principal quantum number (see, e.g., ref. [38] and references therein).

$$U_{\text{eff}} = U + U_0 + U_1, \tag{6}$$

where U is the unperturbed potential energy.

In ref. [2], where the authors studied a hydrogen Rydberg atom in a linearly polarized high-frequency laser field, it was shown that the effective potential energy has the following form:

$$U_{\text{eff}}(r, \theta) = -\frac{e^2}{r} - \frac{\gamma}{r^3}(3\cos^2\theta - 1), \quad \gamma = \frac{e^4F^2}{4m_e^2\omega^4}, \tag{7}$$

where θ is the polar angle, that is, the angle between the radius-vector $\mathbf{r}$ of the electron and the z-axis chosen along the vector-amplitude $\mathbf{F}$ of the laser field. This effective potential energy, which is mathematically equivalent to the potential energy of a satellite around the oblate Earth (see, e.g., ref. [39]), has the following property: in the general case, where the unperturbed electron orbit is elliptical, the orbit undergoes simultaneously two precessions. One is the precession of the ellipse in its plane, and another is the precession of the orbital plane about the vector $\mathbf{F}$. Both of the precession frequencies are of the same order of magnitude and are much smaller than the Kepler frequency of the electron.

In our case, the effective potential energy is generally more complicated. Therefore, we limit ourselves by the situation where the unperturbed orbit of the outer electron is circular. In this situation, the precession in the orbital plane loses its meaning, and we deal only with the precession of the orbital plane.

So, we fix r = const, and only the angle θ remains as the dynamic variable. Then, our effective potential energy can be brought to the form (7) with an additional θ-independent form, as follows:

Introducing the functions:

$$f(x) = (1 + 2x)(1 + 2x^2), \quad g(x) = 3 + 6x + 6x^2 + 4x^3, \tag{8}$$

we can rewrite (4) in the following form:

$$\begin{aligned} U_1 = & -\frac{F^2(Z\mu)^3(Z-1)}{4\mu_1^2\omega^4x^3} \\ & \times \left((3\cos^2\theta - 1)\left(1 + g(x)\frac{e^{-2x}}{3(Z-1)}\right)\right. \\ & \left. + \frac{3f(x) - g(x)}{2}\frac{e^{-2x}}{3(Z-1)}\right), \end{aligned} \tag{9}$$

Therefore, the total energy can be represented in the following form:

$$U_{\text{eff}} \approx -\frac{Z-1}{r} + \Delta U_1(r) + \Delta U(r, \cos\theta), \tag{10}$$

where the second term is the energy shift with respect to the unperturbed energy, and the third term is responsible for the precession.

From (9) and (7), we see that the term

$$g(x)\frac{e^{-2x}}{3(Z-1)} \tag{11}$$

is a relative correction to the precession frequency of the orbital plane, and the term

$$\frac{3f(x) - g(x)}{2}\frac{e^{-2x}}{3(Z-1)} = 4x^3\frac{e^{-2x}}{3(Z-1)} \tag{12}$$

corresponds to an energy shift. Taking into account the factor in the beginning of (9), the energy shift is expressed as follows:

$$\delta E = -\frac{F^2(Z\mu)^3}{3\mu_1^2\omega^4}e^{-2x}. \tag{13}$$

For circular orbits, the energy of the outer electron is $E = -(Z-1)/(2r)$, and, using (5), we write the energy shift as follows:

$$\delta E = -\frac{F^2(Z\mu)^3}{3\mu_1^2\omega^4}e^{\frac{\mu Z(Z-1)}{E}}. \tag{14}$$

Figure 1 shows the dependence of the energy shift on the unperturbed electron energy for $Z = 4$, $F = 1$, and $\omega = 100$. The electron Kepler frequency at these energies is expressed as follows:

$$\omega_{\text{K}} = \frac{1}{Z-1}\sqrt{\frac{8|E|^3}{\mu_1}} \tag{15}$$

which is indeed much smaller than the laser frequency.

It is seen that the shift is zero at the zero unperturbed energy and approaches the limit $-F^2(Z\mu)^3/(3\mu_1^2\omega^4)$ as the unperturbed energy increases by the absolute value (see Figure 2).

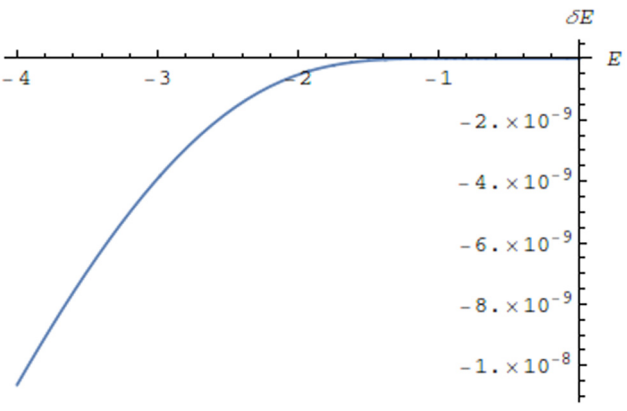

Figure 1: The energy shift versus the unperturbed electron energy for $Z = 4$, $F = 1$, and $\omega = 100$.

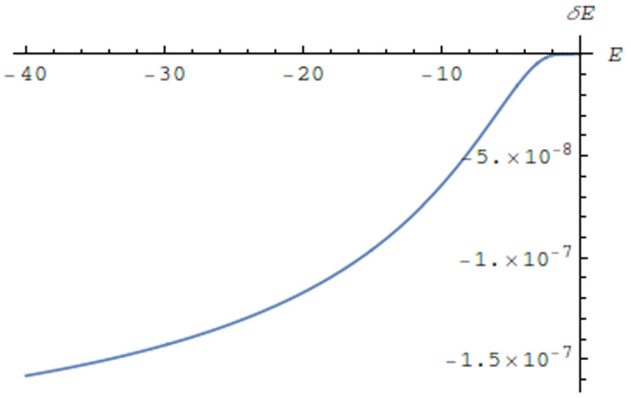

Figure 2: The energy shift versus the unperturbed electron energy E for $Z = 4$, $F = 1$, and $\omega = 100$ for large negative values of E.

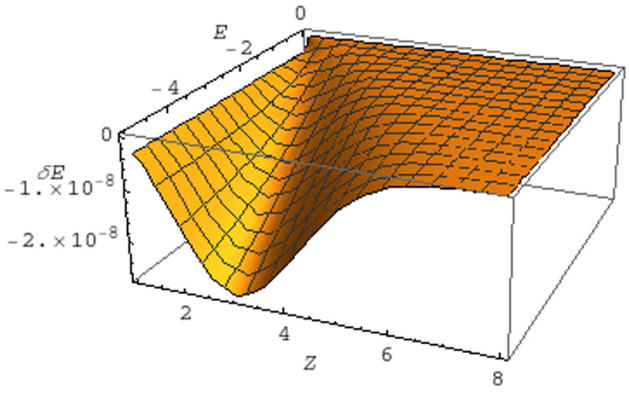

Figure 4: The dependence of the energy shift on Z and E for $F = 1$, $\omega = 100$.

Figure 3 presents the dependence of the energy shift on the nuclear charge Z at $F = 1$ and $\omega = 100$ for two values of the unperturbed energy: $E = -2$ (blue, solid line) and $E = -5$ (red, dashed line). It has a minimum at the point $Z_m \approx (1 + (1 + 24|E|)^{1/2})/4$ (in the approximation $\mu = \mu_1 = 1$). The nonmonotonic dependence of the energy shift on the nuclear charge is a *counterintuitive result*.

Figure 4 shows a 3D plot of the dependence of the energy shift on both the nuclear charge and the unperturbed energy for $F = 1$ and $\omega = 100$. It is seen that the location of the minimum of the energy shift with respect to the nuclear charge indeed moves to higher values of Z as the unperturbed energy increases in the absolute value.

The motion characterized by the effective potential energy can be expressed as follows:

$$U_{\text{eff}} = -\frac{(Z-1)e^2}{r} - \frac{(Z-1)\gamma}{r^3}(3\cos^2\theta - 1) \qquad (16)$$

which is mathematically equivalent to the motion of a satellite around the oblate Earth, and in the latter case,

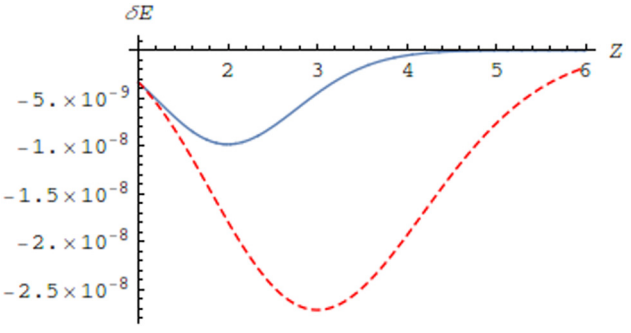

Figure 3: The energy shift versus the nuclear charge Z for $E = -2$ (blue, solid line) and $E = -5$ (red, dashed line) for $F = 1$, $\omega = 100$.

the effective potential (according to ref. [39]) is expressed as follows:

$$V_{\text{E}} = -\frac{GM_{\text{E}}m}{r} - \frac{GM_{\text{E}}m|I_2|R^2}{2r^3}(3\cos^2\theta - 1). \qquad (17)$$

Here M_{E} and m are the Earth and the satellite masses, respectively; R is the equatorial radius of the Earth; and I_2 is a constant characterizing the relative difference between the equatorial and polar diameters of the Earth. The ratio of the precession frequency of the satellite plane Ω to the Kepler frequency ω_{K} is expressed as follows [40]:

$$\frac{\Omega}{\omega_{\text{E,K}}} = \frac{3|I_2|}{2}\left(\frac{R}{p}\right)^2 \cos i, \qquad (18)$$

where

$$\omega_{\text{E,K}} = \sqrt{\frac{G(M_{\text{E}} + m)}{A_{\text{s}}^3}} \qquad (19)$$

is the Kepler frequency of the satellite and m and A_{s} are the satellite mass and the major semi-axis of its unperturbed elliptical orbit, respectively. In equation (18), i is the *inclination*, that is, the angle between the plane of the satellite orbit and the equatorial plane of the Earth. The quantity p is the semi-latus rectum of the unperturbed elliptical orbit.

If in equation (17), we would redefine (i.e., bring into the correspondence)

$$GM_{\text{E}}m = (Z-1)e^2,\ |I_2|R^2 = \frac{2\gamma}{e^2}, \qquad (20)$$

then the effective potential energy from equation (17) would become identical to equation (16). By substituting $|I_2|R^2 = 2\gamma/e^2$ in equation (18), we obtain the corresponding ratio for our case:

$$\left.\frac{\Omega}{\omega_{\mathrm{K}}}\right|_0 = \frac{3\gamma}{e^2 p^2}\cos i, \tag{21}$$

where the Kepler frequency ω_K is given by equation (15) and the index "0" denotes the unperturbed case.

For circular orbits, one has

$$p = r = \frac{Z-1}{2|E|}, \tag{22}$$

so that the ratio from equation (21), corresponding to the unperturbed case, takes the form

$$\left.\frac{\Omega}{\omega_{\mathrm{K}}}\right|_0 = \frac{12\gamma|E|^2}{(Z-1)^2}\cos i, \tag{23}$$

where in our case, the quantity γ is (see equation (9)) expressed as follows:

$$\gamma = \frac{F^2}{4\mu_1^2\omega^4}. \tag{24}$$

For circular orbits, the dependence of the quantity x from equation (23) on the unperturbed energy is expressed as follows:

$$x(E) = \mu Z r = \frac{\mu Z(Z-1)}{2|E|}. \tag{25}$$

In (10), $\Delta U_1(r)$ is a relatively small shift of the energy of the electron. The dynamics of the motion beyond the plane of the unperturbed circular orbit is controlled by the following truncated $U_{\mathrm{eff,tr}}$ (see (9) and (10)):

$$U_{\mathrm{eff,tr}} = -\frac{Z-1}{r} - \frac{(Z-1)\gamma}{r^3} \times \left(1 + g(x)\frac{e^{-2x}}{3(Z-1)}\right)(3\cos^2\theta - 1), \tag{26}$$

where $g(x)$ is given by (8), r is given by (22), and x by (25). Thus, the ratio of the precession frequency to the Kepler frequency of the electron is expressed as follows:

$$\frac{\Omega}{\omega_{\mathrm{K}}} = \frac{3\gamma}{r^2}\left(1 + g(x)\frac{e^{-2x}}{3(Z-1)}\right)\cos i = \frac{12\gamma|E|^2}{(Z-1)^2}\left(1 + g(x)\frac{e^{-2x}}{3(Z-1)}\right)\cos i, \tag{27}$$

and the relative correction to the precession frequency is expressed as follows:

$$\frac{\Delta\Omega}{\Omega} = g(x(E))\frac{e^{-2x(E)}}{3(Z-1)}, \tag{28}$$

with $g(x)$ given in (8) and $x(E)$ given in (25).

Figure 5 presents the dependence of the relative correction to the precession frequency of the orbital plane of the Rydberg electron on the electron energy for selected values of Z.

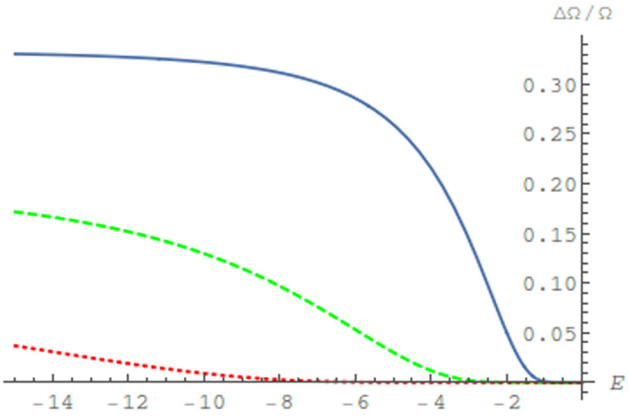

Figure 5: Dependence of the relative correction to the precession frequency of the orbital plane of the Rydberg electron on the electron energy for $Z = 4$ (blue solid curve), $Z = 6$ (green dashed curve), and $Z = 9$ (red dotted curve).

The correction $\Delta\Omega/\Omega$ approaches the limit of $1/(Z-1)$ at large negative values of the electron energy.

2.2 Circular polarization of the laser field

Now we consider the case of the circular polarization of the laser field of the amplitude F and frequency ω, the polarization field being perpendicular to the z-axis. The laser field varies as follows:

$$\mathbf{F} = F(\mathbf{e}_x\cos\omega t + \mathbf{e}_y\sin\omega t), \tag{29}$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are the unit vectors along the x- and y-axes (which are perpendicular to the z-axis). The semiclassical Hamiltonian for the outer (Rydberg) electron in this case can be represented as follows:

$$H = H_0 + xF\cos\omega t + yF\sin\omega t, \tag{30}$$

where H_0 is given in (2). Denoting

$$V = xF = Fr\sin\theta\cos\varphi,\ W = yF = Fr\sin\theta\sin\varphi, \tag{31}$$

where (r, θ, φ) are the spherical coordinates, and using the formalism of the effective potentials [2,6,7], we obtain the zeroth-order effective potential:

$$U_0 = \frac{1}{4\omega^2}([V,[V,H_0]] + [W,[W,H_0]]) = \frac{F^2}{2\mu_1\omega^2} \tag{32}$$

and the first-order effective potential:

$$
\begin{aligned}
U_1 = \frac{1}{4\omega^4}&([[V, H_0], [[V, H_0], H_0]] \\
&+ [[W, H_0], [[W, H_0], H_0]]) \\
&+ \frac{-1}{2\omega^3}[[V, H_0], [W, H_0]] \\
= \frac{F^2(Z\mu)^3}{8\mu_1^2\omega^4 x^3}&((Z-1)(1+3\cos 2\theta) \\
&+ ((1+2x+2x^2-4x^3) \\
&+ (3+6x+6x^2+4x^3)\cos 2\theta)e^{-2x}),
\end{aligned}
\tag{33}
$$

where x is given in (5). By using the same procedure as in the case of the linear polarization, we can rewrite U_1 in the form as in (9):

$$
\begin{aligned}
U_1 = &\frac{F^2(Z\mu)^3(Z-1)}{4\mu_1^2\omega^4 x^3} \\
&\times \left((3\cos^2\theta - 1)\left(1 + g(x)\frac{e^{-2x}}{3(Z-1)}\right)\right. \\
&+ \left.\frac{3f_1(x) - g(x)}{2}\frac{e^{-2x}}{3(Z-1)}\right).
\end{aligned}
\tag{34}
$$

It differs from (9) by the sign in the beginning and a different function $f_1(x)$, which is given by

$$
f_1(x) = 1 + 2x + 2x^2 - 4x^3 \tag{35}
$$

From (34), we find that the energy shift in the case of circular polarization is expressed as follows:

$$
\delta E = -\frac{2F^2(Z\mu)^3}{3\mu_1^2\omega^4}e^{-2x}, \tag{36}
$$

which is twice as much as in the linear case. The relative correction to the precession frequency of the orbital plane is the same as in the linear polarization case (given in (11) and (28)).

3 Conclusions

We studied a helium atom or a helium-like ion with one of the two electrons in a Rydberg state, while the system is subjected to a high-frequency laser field. For obtaining analytical results, we use the generalized method of the effective potentials from the previous works [2,6,7].

Then, we concentrated on circular Rydberg states. We found two primary effects of the high-frequency laser field. The first effect is the precession of the orbital plane of the Rydberg electron. We calculated analytically the precession frequency and showed that it differs from the case of a hydrogenic atom/ion. In the radiation spectrum, this precession would manifest as satellites separated from the spectral line at the Kepler frequency by multiples of the precession frequency.

We also demonstrated that the high-frequency laser field causes a red shift of the energy of the Rydberg electron – the shift that we calculated analytically. We studied its dependence on parameters of the system. We found that the absolute value of the shift increases monotonically as the unperturbed binding energy of the Rydberg electron increases. We also found that the shift has a nonmonotonic dependence on the nuclear charge Z: as Z increases, the absolute value of the shift first increases, then reaches a maximum, and then decreases. The nonmonotonic dependence of the laser field-caused energy shift on the nuclear charge is a *counterintuitive* result.

Finally, we note that for the interaction of a high-frequency laser field with atoms, the formalism of the effective potentials [4–6], employed in this article, has advantages over the formalism developed by Kramers [41] and Henneberger [42] and was later applied by Gavrila and his coauthors to hydrogen atoms [43,44]. The essence of the latter formalism is the transition to the frame oscillating together with an electron in the laser field and then utilizing the time average of the corresponding time-dependent perturbation. However, first, the application of this formalism to hydrogen atoms in the high-frequency laser field, as in the study by Pont and Gavrila [43], misses the hidden (algebraic) symmetry of this system, which is revealed in the previous articles [1,2] via the formalism of the effective potentials; thus, the Kramers–Henneberger formalism lacks the physical insight compared to the formalism of the effective potentials. Second, the formalism of the effective potentials has been developed analytically to the arbitrary order with respect to the laser field [6,7], but this does not seem to be possible for the Kramers–Henneberger formalism.

Conflict of interest: Eugene Oks is an Editor of Open Physics and was not involved in the review process of this article.

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