Useful Formulas

Half argument

$$\sin^2\frac{x}{2}=\frac{1-\cos x}{2}$$
$$\cos^2\frac{x}{2}=\frac{1+\cos x}{2}$$
$$\mathrm{tg}\,\frac{x}{2}=\frac{\sin x}{1+\cos x}=\frac{1-\cos x}{\sin x}$$

Universal substitute

$$\sin 2\mathrm{x}=\frac{2\,\mathrm{tg}\,x}{1+\mathrm{tg}^2 x}$$
$$\cos 2\mathrm{x}=\frac{1-\mathrm{tg}^2 x}{1+\mathrm{tg}^2 x}$$
$$\mathrm{tg}\,2\mathrm{x}=\frac{2\,\mathrm{tg}\,x}{1-\mathrm{tg}^2 x}$$

Inverse hype func

$$\mathrm{Arsh}\,x=\ln(x+\sqrt{x^2+1})$$
$$\mathrm{Arch}\,x=\ln(x\pm\sqrt{x^2-1})$$
$$\mathrm{Arth}\,x=\frac{1}{2}\ln\frac{1+x}{1-x}$$

$$\sin x\pm\sin y=2\sin\frac{x\pm y}{2}\cos\frac{x\mp y}{2}$$
$$\cos x+\cos y=2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\cos x-\cos y=-2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$
$$\mathrm{tg}\,x\pm\mathrm{tg}\,y=\frac{\sin(x\pm y)}{\cos x\cos y}$$

$$\sin x\sin y=\frac{1}{2}(\cos(x-y)-\cos(x+y))$$
$$\cos x\cos y=\frac{1}{2}(\cos(x-y)+\cos(x+y))$$
$$\sin x\cos y=\frac{1}{2}(\sin(x-y)+\sin(x+y))$$

Curvature radius

$$R=\frac{(x'^2+y'^2)^{3/2}}{|x'\,y''-y'\,x''|}$$
$$R=\frac{(1+y'^2)^{3/2}}{y''}$$

f(x) figure area

$$S=\int_a^b f(x)\,dx$$
$$S=\left|\int_{t_1}^{t_3}\phi(t)\psi'(t)\,dt\right|$$
$$S=\int_{\phi_1}^{\phi_2} r^2(\phi)\,d\phi$$

f(x) curve length

$$L=\int_a^b\sqrt{1+f'^2(x)}\,dx$$
$$L=\int_\alpha^\beta\sqrt{\phi'^2(t)+\psi'^2(t)}\,dt$$
$$L=\int_\alpha^\beta\sqrt{r^2(\phi)+r'^2(\phi)}\,d\phi$$

f(x) rotation body parameters

$$V_{Ox}=\pi\int_a^b f^2(x)\,dx$$
$$S_{Ox}=\int_a^b 2\pi f\sqrt{1+f^2}\,dx$$

Newton's binomial

$$(x+a)^n=\sum_{k=0}^{n} C_n^k x^{n-k} a^k$$

Binomial differential

$$\int x^m(a+bx^n)^p\,dx$$

1) $p\in Z:\ x=t^N,\ N=com.div.(m,n)$
2) $\frac{m+1}{n}\in Z:\ a+bx^n=t^N,\ N=div.(p)$
3) $\frac{m+1}{n}+p\in Z:\ ax^{-n}+b=t^N,\ N=div.(p)$