Таблица интегралов

(a, b, m, n – постоянные)

I. Интегралы от рациональных функций

1. $\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C,$ n – целое число, не равное 1.
2. $\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b| + C = \frac{1}{a}\ln C|ax+b|.$
3. $\int \frac{xdx}{ax+b} = \frac{1}{a^2}\left[ax - b\ln(ax+b)\right] + C.$
4. $\int \frac{x^2 dx}{ax+b} = \frac{x^2}{2a} - \frac{bx}{a^2} + \frac{b^2}{a^3}\ln(ax+b) + C.$
5. $\int \frac{dx}{x(ax+b)} = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right| + C.$
6. $\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right| + C.$
7. $\int \frac{xdx}{(x+b)^2} = \frac{1}{a^2}\left(\ln|ax+b| + \frac{b}{ax+b}\right) + C.$
8. $\int \frac{x^2 dx}{(x+b)^2} = \frac{1}{a^3}\left[ax - 2b\ln|ax+b| - \frac{b^2}{ax+b}\right] + C.$
9. $\int \frac{dx}{x(ax+b)^2} = \frac{1}{b(ax+b)} - \frac{1}{b^2}\ln\left|\frac{ax+b}{x}\right| + C.$
10. $\int \frac{dx}{x^2+b^2} = \frac{1}{ab}\operatorname{arctg}\frac{a}{b}x + C.$
11. $\int \frac{dx}{x^2-b^2} = \frac{1}{2ab}\ln\left|\frac{ax-b}{ax+b}\right| + C.$
12. $\int \frac{xdx}{x^2 \pm b^2} = \frac{1}{2a^2}\ln\left|a^2x^2 \pm b^2\right| + C.$
13. $\int \frac{x^2 dx}{x^2+b^2} = \frac{1}{a^2}x - \frac{b}{a^3}\operatorname{arctg}\frac{a}{b}x + C.$
14. $\int \frac{x^2 dx}{x^2-b^2} = \frac{1}{a^2}x + \frac{b}{2a^3}\ln\left|\frac{ax-b}{ax+b}\right| + C.$
15. $\int \frac{dx}{x(a^2x^2+b^2)} = \frac{1}{2b^2}\ln\frac{x^2}{a^2x^2+b^2} + C.$
16. $\int \frac{dx}{x(a^2x^2-b^2)} = \frac{1}{2b^2}\ln\left|\frac{a^2x^2-b^2}{x^2}\right| C.$
17. $\int \frac{dx}{x^2(a^2x^2+b^2)} = -\frac{1}{b^2x} - \frac{a}{b^3}\operatorname{arctg}\frac{a}{b}x + C.$
18. $\int \frac{dx}{x^2(a^2x^2-b^2)} = \frac{1}{b^2x} + \frac{a}{2b^3}\ln\left|\frac{ax-b}{ax+b}\right| + C.$

II. Интегралы от иррациональных функций

19. $$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \quad n \neq -1.$$

20. $$\int \sqrt{ax+b}\,dx = \frac{2}{15a^2}(3ax-2b)\sqrt{(ax+b)^3} + C.$$

21. $$\int x^2\sqrt{ax+b}\,dx = \frac{2}{105a^3}\left(15a^2x^2 - 12abx + 8b^2\right)\sqrt{(ax+b)^3} + C.$$

22. $$\int \frac{x}{\sqrt{x+b}}\,dx = \frac{2(ax-2b)}{3a^2}\sqrt{ax+b} + C.$$

23. $$\int \frac{x^2}{\sqrt{x+b}}dx = \frac{2}{15a^3}\left(3a^2x^2 - 4abx + 8b^2\right)\sqrt{ax+b} + C.$$

24. $$\int \frac{dx}{x\sqrt{ax+b}} \begin{cases} = \frac{1}{\sqrt{b}}\ln\frac{\sqrt{ax+b}-\sqrt{b}}{\sqrt{ax+b}+\sqrt{b}} + C, & b > 0, \\ = \frac{2}{\sqrt{-b}}\operatorname{arctg}\sqrt{\frac{ax+b}{-b}} + C, & b < 0. \end{cases}$$

25. $$\int \frac{dx}{\sqrt{ax+b}} = -\frac{\sqrt{ax+b}}{bx} - \frac{a}{2b}\int \frac{dx}{x\sqrt{ax+b}}.$$

26. $$\int \frac{\sqrt{ax+b}}{x}dx = 2\sqrt{ax+b} + b\int \frac{dx}{x\sqrt{ax+b}}.$$

27. $$\int \frac{\sqrt{ax+b}}{x^2}dx = -\frac{\sqrt{ax+b}}{x} + \frac{a}{2}\int \frac{dx}{x\sqrt{ax+b}}.$$

В формулах 28 – 47 считается, что $a > 0$, $b > 0$.

28. $$\int \frac{dx}{\sqrt{a^2x^2 \pm b^2}} = \frac{1}{a}\ln\left|ax + \sqrt{a^2x^2 \pm b^2}\right| + C.$$

29. $$\int \frac{dx}{\sqrt{b^2 - a^2x^2}} = \frac{1}{a}\arcsin\frac{ax}{b} + C.$$

30. $$\int \frac{xdx}{\sqrt{a^2x^2 \pm b^2}} = \frac{1}{a^2}\sqrt{a^2x^2 \pm b^2} + C.$$

31. $$\int \frac{xdx}{\sqrt{b^2 - a^2x^2}} = -\frac{1}{a^2}\sqrt{b^2 - a^2x^2} + C.$$

32. $$\int \frac{x^2dx}{\sqrt{a^2x^2 \pm b^2}} = \frac{1}{2a^3}\left[ax\sqrt{a^2x^2 \pm b^2} \mp b^2\ln\left|ax + \sqrt{a^2x^2 \pm b^2}\right|\right] + C.$$

33. $$\int \frac{x^2dx}{\sqrt{b^2 - a^2x^2}} = \frac{1}{2a^3}\left[-ax\sqrt{b^2 - a^2x^2} + b^2\arcsin\frac{ax}{b}\right] + C.$$

34. $$\int \sqrt{a^2x^2 \pm b^2}\,dx = \frac{1}{2}\left[x\sqrt{a^2x^2 \pm b^2} \pm \frac{b^2}{a}\ln\left|ax + \sqrt{a^2x^2 \pm b^2}\right|\right] + C.$$

35. $$\int \sqrt{b^2 - a^2x^2}\,dx = \frac{1}{2}\left[x\sqrt{b^2 - a^2x^2} + \frac{b^2}{a}\arcsin\frac{ax}{b}\right] + C.$$

36. $$\int \sqrt{a^2x^2 \pm b^2}\,dx = \frac{1}{3a^2}\sqrt{\left(a^2x^2 \pm b^2\right)^3} + C.$$

37. $$\int \sqrt{b^2 - a^2x^2}\,dx = -\frac{1}{3a^2}\sqrt{\left(b^2 - a^2x^2\right)^3} + C.$$

38. $$\int x^2\sqrt{a^2x^2 \pm b^2}\,dx = \frac{1}{8a^3}\left[ax\left(2a^2x^2 \pm b^2\right)\sqrt{a^2x^2 \pm b^2} - b^4 \ln\left|ax + \sqrt{a^2x^2 \pm b^2}\right|\right] + C.$$

39. $$\int x^2\sqrt{b^2 - a^2x^2}\,dx = \frac{1}{8a^3}\left[ax\left(2a^2x^2 - b^2\right)\sqrt{b^2 - a^2x^2} + b^4 \arcsin\frac{ax}{b}\right] + C.$$

40. $$\int \frac{\sqrt{a^2x^2 + b^2}}{x}dx = \sqrt{a^2x^2 + b^2} + \frac{b}{2}\ln\frac{\sqrt{a^2x^2 + b^2} - b}{\sqrt{a^2x^2 + b^2} + b} + C$$

Примечание[:1)] *Правой части можно придать вид на основании тождества*

$$\frac{\sqrt{a^2x^2 + b^2} - b}{x^2 + b^2 + b} = \left(\frac{ax}{\sqrt{a^2x^2 + b^2} + b}\right)^2 = \left(\frac{\sqrt{a^2x^2 + b^2} - b}{ax}\right)^2.$$

40'. $$\int \frac{\sqrt{a^2x^2 - b^2}}{x}dx = \sqrt{a^2x^2 - b^2} + b\arcsin\left|\frac{b}{ax}\right| + C.$$

41. $$\int \frac{\sqrt{b^2 - a^2x^2}}{x}dx = \sqrt{b^2 - a^2x^2} - b\ln\left|\frac{b - \sqrt{b^2 - a^2x^2}}{x}\right| + C.$$

42. $$\int \frac{\sqrt{a^2x^2 \pm b^2}}{x^2}dx = -\frac{\sqrt{a^2x^2 \pm b^2}}{x} + a\ln\left|ax + \sqrt{a^2x^2 \pm b^2}\right| + C.$$

43. $$\int \frac{\sqrt{b^2 - a^2x^2}}{x^2}dx = -\frac{\sqrt{b^2 - a^2x^2}}{x} - a\arcsin\frac{ax}{b} + C.$$

44. $$\int \frac{dx}{\sqrt{a^2x^2 + b^2}} = \frac{1}{b}\ln\left|\frac{x}{b + \sqrt{a^2x^2 + b^2}}\right| + C.$$

44'. $$\int \frac{dx}{\sqrt{a^2x^2 - b^2}} = -\frac{1}{b}\arcsin\left|\frac{b}{ax}\right| + C.$$

45. $$\int \frac{dx}{\sqrt{b^2 - a^2x^2}} = \frac{1}{b}\ln\left|\frac{x}{b + \sqrt{b^2 - a^2x^2}}\right| + C.$$

46. $$\int \frac{dx}{\sqrt{a^2x^2 \pm b^2}} = \mp\frac{\sqrt{a^2x^2 \pm b^2}}{b^2x} + C.$$

47. $$\int \frac{dx}{\sqrt{b^2 - a^2x^2}} = -\frac{\sqrt{b^2 - a^2x^2}}{b^2x} + C.$$

48. $$\int \frac{\sqrt{a + x}}{b + x}dx = \sqrt{(a + x)(b + x)} + (a - b)\ln\left(\sqrt{a + x} + \sqrt{b + x}\right) + C.$$

49. $$\int \frac{\sqrt{a - x}}{b + x}dx = \sqrt{(a - x)(b + x)} + (a + b)\arcsin\sqrt{\frac{x + b}{a + b}} + C.$$

50. $$\int \frac{\sqrt{a + x}}{b - x}dx = -\sqrt{(a + b)(b - x)} - (a + b)\arcsin\sqrt{\frac{b - x}{a + b}} + C.$$

51. $$\int \frac{\sqrt{1 + x}}{1 - x}dx = -\sqrt{1 - x^2} + \arcsin x + C.$$

52. $$\int \frac{dx}{\sqrt{(x - a)(x - b)}} = \ln\left|\frac{\sqrt{x - a} + \sqrt{x - b}}{\sqrt{x - a} - \sqrt{x - b}}\right| + C.$$

53. $$\int \frac{dx}{\sqrt{(x-a)(b-x)}} = 2\arcsin\sqrt{\frac{x-a}{b-a}} + C.$$

54. $$\int \operatorname{sh} x\,dx = \operatorname{ch} x + C.$$

m, n, r, s – целые числа, $s > 1$. Применяется подстановка $ax^r + b = u^s$, если $\frac{m+1}{r}$ – число целое, и подстановка $ax^{-r} + b = u^s$, если $\frac{m+1}{r} + \frac{n}{s}$ – число целое. В других случаях интеграл не выражается элементарной функцией.

III. Интегралы от трансцендентных функций

55. $$\int x^n e^x dx = e^x\left[x^n - nx^{n-1} + n(n-1)x^{n-2} - \ldots + (-1)^n n!\right] + C,$$ n – целое положительное число.

56. $$\int (\ln x)^n dx = x\left[(\ln x)^n - n(\ln x)^{n-1} + n(n-1)(\ln x)^{n-2} - \ldots + (-1)^n n!\right] + C,$$ n – целое положительное число.

57. $$\int e^{ax} \sin nx\,dx = \frac{e^{ax}(a\sin nx - n\cos nx)}{a^2 + n^2} + C.$$

58. $$\int \frac{(\ln x)^n}{x} dx = \frac{(\ln x)^{n+1}}{n+1} + C, \quad n \neq -1.$$

59. $$\int \frac{dx}{x\ln x} = \ln|\ln x| + C.$$

60. $$\int \cos nx\,dx = -\frac{\cos nx}{n} + C.$$

61. $$\int \sin nx\,dx = \frac{\sin nx}{n} + C.$$

62. $$\int x\cos nx\,dx = -\frac{\cos(m+n)x}{2(m+n)} - \frac{\cos(m-n)x}{2(m-n)} + C, \quad m \neq n.$$

63. $$\int x\sin nx\,dx = \frac{\sin(m-n)x}{2(m-n)} - \frac{\sin(m+n)x}{2(m+n)} + C, \quad m \neq n.$$

64. $$\int \sin mx\cos nx\,dx = \frac{\sin(m-n)x}{2(m-n)} + \frac{\sin(m+n)x}{2(m+n)} + C, \quad m \neq n.$$

65. $$\int x\cos x\,dx = \frac{1}{2}\sin^2 x + C = -\frac{1}{2}\cos^2 x + C.$$

66. $$\int \sin^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C.$$

67. $$\int \cos^2 x\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C.$$

68. $$\int \frac{dx}{\sin x} = \ln\left|\operatorname{tg}\frac{x}{2}\right| + C.$$

69. $$\int \frac{dx}{\cos x} = \ln\left|\operatorname{tg}\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C.$$

70. $$\int \sin^m x\cos^n x\,dx \begin{cases} = -\dfrac{\sin^{m-1} x\cos^{n+1} x}{m+n} + \dfrac{m-1}{m+n}\displaystyle\int \sin^{m-2} x\cos^n x\,dx, & m \neq -n, \\ = \dfrac{\sin^{m+1} x\cos^{n-1} x}{m+n} + \dfrac{n-1}{m+n}\displaystyle\int \sin^m x\cos^{n-2} x\,dx, & m \neq -n. \end{cases}$$

71. $$\int \frac{dx}{\sin^m x\cos^n x} \begin{cases} = -\dfrac{1}{m-1}\dfrac{1}{\sin^{m-1} x\cos^{n-1} x} + \dfrac{m+n-2}{m-1}\displaystyle\int \frac{dx}{\sin^{m-2} x\cos^n x}, & m \neq 1, \\ = \dfrac{1}{n-1}\dfrac{1}{\sin^{m-1} x\cos^{n-1} x} + \dfrac{m+n-2}{n-1}\displaystyle\int \frac{dx}{\sin^m x\cos^{n-2} x}, & n \neq 1. \end{cases}$$

72. $\int \mathrm{tg}^n x\,dx = \frac{\mathrm{tg}^{n-1} x}{n-1} - \int \mathrm{tg}^{n-2} x\,dx, \quad n \neq 1.$

73. $\int \mathrm{ctg}^n x\,dx = -\frac{\mathrm{ctg}^{n-1} x}{n-1} - \int \mathrm{ctg}^{n-2} x\,dx, \quad n \neq 1.$

74. $\int x^n \sin x\,dx = -x^n \cos x + n\int x^{n-1} \cos x\,dx.$

75. $\int x^n \cos x\,dx = x^n \sin x - n\int x^{n-1} \sin x\,dx.$

76. $$\int \frac{dx}{a + b\cos x} \begin{cases} = \frac{2}{\sqrt{a^2 - b^2}} \mathrm{arctg}\left(\sqrt{\frac{a-b}{a+b}}\,\mathrm{tg}\frac{x}{2}\right) + C, & a^2 > b^2, \\ = \frac{1}{\sqrt{b^2 - a^2}} \ln\left|\frac{\sqrt{b^2 - a^2}\,\mathrm{tg}\frac{x}{2} + a + b}{\sqrt{b^2 - a^2}\,\mathrm{tg}\frac{x}{2} - a - b}\right| + C, & a^2 < b^2. \end{cases}$$

77. $$\int \frac{dx}{a + b\sin x} \begin{cases} = \frac{2}{\sqrt{a^2 - b^2}} \mathrm{arctg}\frac{a\,\mathrm{tg}\frac{x}{2} + b}{\sqrt{a^2 - b^2}} + C, & a^2 > b^2, \\ = \frac{1}{\sqrt{b^2 - a^2}} \ln\left|\frac{a\,\mathrm{tg}\frac{x}{2} + b - \sqrt{b^2 - a^2}}{a\,\mathrm{tg}\frac{x}{2} + b + \sqrt{b^2 - a^2}}\right| + C, & a^2 < b^2. \end{cases}$$

78. $\int \frac{dx}{a\cos x + b\sin x} = \frac{1}{\sqrt{a^2 + b^2}} \ln\left|\frac{a\,\mathrm{tg}\frac{x}{2} - b + \sqrt{a^2 + b^2}}{a\,\mathrm{tg}\frac{x}{2} - b - \sqrt{a^2 + b^2}}\right|.$

В частности, $\int \frac{dx}{\cos x + \sin x} = \frac{1}{\sqrt{2}} \ln\left|\mathrm{tg}\left(\frac{x}{2} + \frac{\pi}{8}\right)\right|.$

79. $\int \frac{dx}{\sin^2 x + b^2 \cos^2 x} = \frac{1}{ab} \mathrm{arctg}\left(\frac{a\,\mathrm{tg}\,x}{b}\right) + C.$

80. $\int e^{ax} \sin nx\,dx = \frac{e^{ax}(a\sin nx - n\cos nx)}{a^2 + n^2} + C.$

81. $\int e^{ax} \cos nx\,dx = \frac{e^{ax}(n\sin nx + a\cos nx)}{a^2 + n^2} + C.$

82. $\int e^{ax} \sin^n x\,dx = \frac{e^{ax} \sin^{n-1} x(a\sin x - n\cos x)}{a^2 + n^2} + \frac{n(n-1)}{a^2 + n^2}\int e^{ax} \sin^{n-2} x\,dx.$

83. $\int e^{ax} \cos^n x\,dx = \frac{e^{ax} \cos^{n-1} x(a\cos x + n\sin x)}{a^2 + n^2} + \frac{n(n-1)}{a^2 + n^2}\int e^{ax} \cos^{n-2} x\,dx.$

84. $\int \sin x\,dx = x\,\mathrm{arcsin}\,x + \sqrt{1 - x^2} + C.$

85. $\int \cos x\,dx = x\,\mathrm{arccos}\,x - \sqrt{1 - x^2} + C.$

86. $\int \mathrm{tg}\,x\,dx = x\,\mathrm{arctg}\,x - \frac{1}{2}\ln(1 + x^2) + C.$

87. $\int x\,\mathrm{arc}\sin x\,dx = \left(\frac{x^2}{2} - \frac{1}{4}\right)\mathrm{arc}\sin x + \frac{x\sqrt{1 - x^2}}{4}.$

88. $\int x \arccos x dx = \left(\frac{x^2}{2} - \frac{1}{4}\right)\arccos x - \frac{x\sqrt{1-x^2}}{4}.$

89. $\int \operatorname{arctg} x dx = \frac{1}{2}\left(x^2+1\right)\operatorname{arctg} x - \frac{x}{2}.$

90. $\int \operatorname{sh} x dx = \operatorname{ch} x + C.$

91. $\int \operatorname{ch} x dx = \operatorname{sh} x + C.$

92. $\int \operatorname{th} x dx = \ln \operatorname{ch} x + C.$

93. $\int \operatorname{sh}^2 x dx = -\frac{1}{2}x + \frac{1}{4}\operatorname{sh} 2x + C.$

94. $\int \operatorname{ch}^2 x dx = \frac{1}{2}x + \frac{1}{4}\operatorname{ch} 2x + C.$