Final Exam - Fall 2008 - Take Home

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1. (5 points) (Winter 1997 GDE)

■ Problem

A conducting sphere of radius b is placed at the origin. A uniformly charged ring of radius a, total charge q, and mass m is free to move along the z-axis outside the sphere with its center on the z-axis and the plane of the ring perpendicular to the z-axis. No force other than electric is present.

■ (a)

What is the potential that you must apply to the sphere to make the force on the ring vanish at $z = z_o$?

■ (b)

Obtain the limiting form of the potential when $a \to 0$.

■ (c)

Does the potential applied to the conducting sphere match the value you anticipate in this limit?

■ (d)

If the potential on the sphere is fixed at the value which makes the force on the charge q vanish at $z = z_o$, what happens to the point charge q when it is displaced from z_o?

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2. (5 points) (Winter 1997 GDE)

■ Problem

A long, thin conducting cylindrical shell of length L , radius a, and mass m is rotating around its axis with a rotational speed ω. A total charge Q is carefully transferred to the rotating cylinder without friction. Figure out the new rotational speed of the cylinder, after the charge Q is placed on it.

3. (5 points) (old GDE)

Problem

A plasma (ionized gas) is compressible and has a pressure which can be written nkT, where n is the particle density, k is the Boltzmann constant and T is the absolute temperature. It is also a conductor which can carry an electric current which in turn creates a magnetic field. Consider a cylindrical plasma column of initial radius $a_o$carrying a current $I_o$in the $+z$ direction. If the current is established very quickly, it flows only on the surface of the column.

(a)

Use the Maxwell stress tensor to derive an expression for equilibrium between the particle pressure and the magnetic field pressure across the surface of the column.

(b)

What is the critical current above which the column will be compressed by its own magnetic field?

(c)

What happens if the current remains above this value?

4. (5 points) (old GDE)

Problem

A parallel plate capacitor consists of two coaxial circular plates of radius a, separated by a small gap d $(d << a)$. Charges $+q_o$and $-q_o$are placed on the two plates, and, at time $t = 0$, their centers are connected with a thin straight wire of radius b $(b << d)$ and a resistance R. Assume that R is very large, so that at any time, the field across the plates remains uniform, and the inductance can be neglected. Also, neglect any fringing effect of the electric field at the edge of the plates.

(a)

Calculate the charge on the capacitor plate as a function of time.

(b)

Calculate the magnetic field between the capacitor plates as a function of time.

(c)

Calculate the Poynting vector at radius b and at radius a. Comment on the interpretation of these results.

5. (5 points) (old GDE)

Problem

A very long solenoid of radius a and N turns per unit length carries a current I_{sol}. Coaxial with the solenoid is a circular ring of wire of radius b $(b >> a)$ and electrical resistance R. When the current in the solenoid is gradually decreased, a current I_r is induced in the ring.

(a)

Calculate I_r as a function of $\frac{dI_{sol}}{dt}$.

(b)

The power delivered to the ring (and dissipated in it by resistive losses) comes from the solenoid.

i

Explicitly calculate the Poynting vector just outside the solenoid. (Why is there an electric field outside the solenoid? Why is there a magnetic field outside the solenoid?)

ii

Show that energy is conserved, i.e., that the integrated Poynting flux from the solenoid is equivalent to the power dissipated in the ring.

6. (5 points) (Fall 1996 GDE)

An initially uniform electric field E_o is directed along the z-axis, and the potential at the origin is U_o. In this field is placed a dielectric sphere (dielectric constant = κ) centered on the origin with a total charge Q distributed uniformly over its surface. The radius of the sphere is R. Find the electrostatic potential inside and outside the sphere.