Newton's Laws

A measure of interaction between two objects is called a *force*. Forces are vector quantities, therefore, they can be added according to vector addition rules. The second Newton's law states that the total (net) force on any object is proportional to its acceleration, and the coefficient of proportionality is called *mass*. In mathematical notation,

$$\vec{F} = m\vec{a}$$

That is, mass is a measure of the object's reaction to interaction: the bigger the mass, the more reluctant it is to change its motion under an interaction.

Particularly, when the acceleration is zero, any object with a mass must have a zero acceleration, that is, either rest or move with a constant velocity. A constant force will make it move with a constant acceleration. This may be a little counterintuitive with respect to everyday experience: if you drag a box with a constant force, it seems to move uniformly, without acceleration. In fact, this uniform motion is caused by the balance of the dragging force and the friction force: you need to drag it exactly with the same force as friction (observe how hard it is to drag it on a rough wooden floor and how easy it is on slippery ice); in any case, the dragging force and the friction force are equal in magnitude and opposite in direction and therefore both vectors sum up to zero. If there were no friction at all, once pushed, the box would move uniformly until it hits an obstacle.

It would be worth mentioning first and third Newton's laws: the first states that without any interaction, any object will either rest or keep constant velocity (this actually follows from the second law with zero acceleration); the third states that in any two-body interaction, the forces which the two exert on each other are equal in magnitude, opposite in direction, and lie on the same line.

Any typical Newton's law problem can be solved in the following steps.

1. Identify all forces acting on the object under observation and draw them. Identify the object's acceleration and draw it (if present).
2. Choose the coordinate system. Since you need only to project forces, it doesn't matter where you put your origin. Also, it is always convenient to direct one of the axes at the acceleration so that you would have zero acceleration on the other (perpendicular) axis. (Everything here is unlike in kinematics where the position of the origin matters and it is best to leave the axes horizontal and vertical.)
3. Project the vector equation $m\mathbf{a} = \mathbf{F}_{net}$, which is second Newton's law, on both axes. You will get two equations.
4. If these two equations are enough for solving for the answer (i.e. there are only two unknowns in the system of equations), solve for the answer.

Do not confuse the acceleration g with the force of gravity mg! When the object is free-falling and the only force acting on it is $m\mathbf{g}$, its acceleration (obviously from common experience and from the second Newton's law) will be $\mathbf{g}$. When it is not free-falling, i.e., there are forces other than gravity acting on it, its acceleration can be anything depending on the situation (whether it is hanging, resting on a horizontal surface, incline plane, with or without friction, etc.), but the force of gravity will always be there, unless the object is removed from planet Earth.

Example problem 1. Two unequal masses m_1 and m_2 are attached to the ends of an ideal (massless and impossible to stretch) string and suspended from an ideal (massless) pulley as shown on Fig.1. What is the acceleration of each mass? What is the tension of the string and of the cord that supports the pulley?

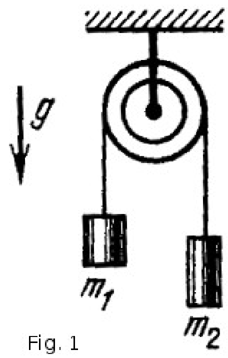

Fig. 1

Solution. Let's identify and draw all the forces and accelerations acting on the two masses and the pulley (Fig. 2).

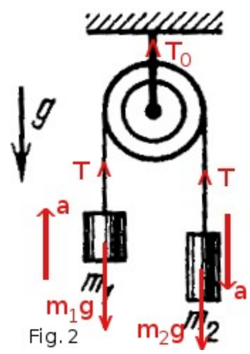

Fig. 2

First of all, we assumed that the right mass is heavier – clearly the direction of acceleration depends on which mass is heavier, and we choose the direction in which we count it as positive; if it is the other way, it will come out negative. (By the way, an important conclusion you should make regarding the answer for *a*: since the sign of the acceleration is determined by whether m_1 is greater or less than m_2, the answer for *a* should contain m_1 - m_2.) Second, from the fact that the string is non-stretchable it follows that the accelerations of both masses are equal in magnitude – if it were, say, a rubber string, one of them could hang still while the other would be moving; that's why we used the same variable for both. Third, the tension in the string is also the same all throughout the string due to its being massless. Just see: if the tensions were different, there would be a non-zero net force acting on the *string*; however, as it has a zero mass, by the second Newton's law the net force on it must also be zero.

Projecting the left mass on the vertical upward axis, we have $m_1 a = T - m_1 g$.

Projecting the right mass on the vertical downward axis, we have $m_2 a = m_2 g - T$.

Thus, we have a system of two equations with two unknowns *a* and *T*, both of which are required to find. By adding the two equations together, we have $(m_1 + m_2) a = (m_2 - m_1) g$, and

$$a = \frac{m_2 - m_1}{m_1 + m_2} g \quad ,$$

and, as predicted, it is proportional to the difference of the masses. Now divide the equations by each other (or substitute the found value of *a* to either of the equations) to find *T*:

$$\frac{m_1}{m_2} = \frac{T - m_1 g}{m_2 g - T}, m_1 m_2 g - m_1 T = m_2 T - m_1 m_2 g, (m_1 + m_2) T = 2 m_1 m_2 g \quad \text{, and}$$

$$T = \frac{2 m_1 m_2 g}{m_1 + m_2} \quad .$$

Looking at the answers for *a* and *T*, we see that if any of the masses is zero, the tension is also zero, and the acceleration of the other mass will be *g*. This agrees with experiment, because if you remove one of the masses (or make it extremely lightweight), the other will just free-fall and drag the string with itself which will not have any tension. In another easy-to-see situation when both masses are equal, we must expect zero acceleration and the tension equal to the weight of either mass. Indeed, for equal masses the formula for *a* yields zero and the formula for *T* yields $2m^2g/2m = mg$, where $m = m_1 = m_2$.

What about the tension of the upper cord which supports the pulley? We can find it by using the second Newton's law on the *pulley*. The pulley's acceleration is zero, there is the tension T_0 of the cord acting up and the two tensions equal to *T* acting down. The second Newton's law then will be

$$0 = T_0 - 2T \quad \text{, and} \quad T_0 = 2T = \frac{4 m_1 m_2 g}{m_1 + m_2} \quad .$$

Friction. Many problems on dynamics involve friction. Friction is interaction of an object with a rough surface. In this course static and dynamic friction are studied.

Let's consider an object, say, a box, resting on a rough horizontal surface. If we apply a small horizontal force trying to drag or push it but insufficient for it to start moving, it will stay still. That means there is an equal and opposite force acting on it which prevents its motion. It is called *the force of static friction*. If we increase our pull or push, so will the force of static friction. But this can't go on forever. Sooner or later the bond between the rough surfaces will break and the box will start moving. If we keep pushing it with a constant force and it moves with a constant velocity (and thus zero acceleration), that means our force is equal and opposite to *the force of dynamic friction*.

The dynamic friction force has an easy relationship with the normal force via Coulomb – Amontons law: $F_f = \mu N$, where *N* is the normal force and μ is a number depending on the material of the participating surfaces, called *the coefficient of friction*. For most materials, μ ranges from 0.1 to 0.6. Coefficients of friction equal to 1 or greater indicate very strong friction; a very slippery surface would have a μ about 0.05.

μ is not a force of friction! μ is a dimensionless number which can be used to find the force of friction (by multiplying the normal force by it). This confusion is widespread among Physics 1600 students.

Example problem 2. A box of mass *m* is being dragged with force *F* under angle α on the rough surface with coefficient of friction μ. Provided that the box not leave the surface (it may actually fly up if you pull too hard), find its acceleration.

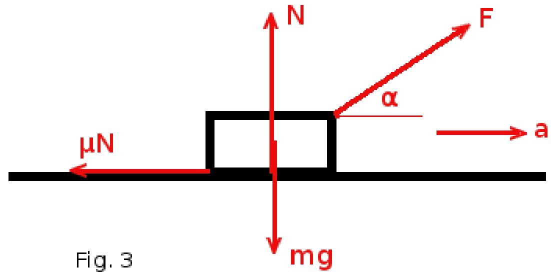

Fig. 3

Solution. Identifying all forces and accelerations as in Fig. 3 and using Coulomb – Amontons law to state that in motion the friction force equals μN, we project second Newton's law on horizontal and vertical axes:

Horizontal: $ma = F\cos\alpha - \mu N$
Vertical: $0 = N + F\sin\alpha - mg$

Excluding the unknown (and unwanted) N from the system by solving the second equation for it and substituting it into the first, we have

$$ma = F\cos\alpha - \mu(mg - F\sin\alpha) = F(\cos\alpha + \mu\sin\alpha) - \mu m g$$, and

$$a = \frac{F}{m}(\cos\alpha + \mu\sin\alpha) - \mu g$$.